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Increasing resilience of electricity networks: Auctioning of priority supply to minimize outage costs

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Abstract:

This article presents an approach to minimize the outage costs during power supply disruptions and, thus, to incentivize efficient resilience investment by network users. The central problem to be solved is the information asymmetry between network operators and network users on outage and backup costs. We present an auction of priority positions among network users based on the Vickrey-Clarke-Groves mechanism, using a numerical example, to solve the problem. Under the mechanism, each winning bidder pays for the externality exerted on the other bidders by holding a certain position, excluding her own bid, which induces truthful bidding. Minimizing the damage from power supply interruptions, the mechanism improves the resilience of the power system not only in the short term but also in the long term.

Keyword: Resilience, electricity network, position auction, priority supply

JEL-classification: D44, K23, L94

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1. Introduction

In recent years, the resilience of the electricity system has become an important issue of security of supply, especially due to the rapid developments in decentralisation and digitalisation as well as natural calamities (c.f. e.g. acatech, 2020; European Council, 2021; Anderson et al., 2019). Resilience is the ability to deal fast and efficiently with potential large-scale and prolonged supply interruptions (c.f. acatech, 2020). One pillar of increasing the resilience of the electricity system is to minimize the damage costs in the case of supply disruptions. Efficiency in handling supply disruption and restoration implies that the network users with the lowest outage costs are disconnected first and reconnected last in an adverse event. Efficient dispatch during supply disruptions is also a prerequisite for efficient investment in backup capacities by network users in the longer term, because they base their decision on the probability of interruption occurrences (c.f. e.g. Anderson et al. 2019). If the expected outage probabilities result from inefficient rationing methods, then backup decisions are also deemed to be inefficient.

To achieve an efficient ranking of network users, network operators, who manage supply disruptions, need to be able to prioritize network users accordingly in the case of an event and to dispose of the cost information. Network operators are typically legally required to treat network users non-discriminatorily, which can be achieved by random or proportional rationing, but still have some leeway in prioritizing certain consumers or consumer groups (c.f. e.g. for Germany, BMWK, 2023). Information asymmetries between network users and network operators are thus the central challenge for achieving an efficient ranking of network users.

One possible approach to address the information asymmetries would be to estimate the costs of supply interruptions, as it is common in quality regulation, expressed as the "Value of Lost Load" (CEER, 2022). The estimated values of lost load vary, however, even within one country with e.g. the chosen method and the geographic granularity of the approach (De Nooij et al., 2007). Further, one would additionally have to estimate the costs of backup measures that may also vary a lot between network users (e.g. switch to other fuels, battery storage, etc.).

In this article, we therefore suggest to address this problem by means of the Vickrey-Clarke-Groves (VCG) mechanism. The VCG mechanism comprises an allocation and a payment rule. More specifically, in this context, network operators auction the priority positions and, based on the reported values, reallocate the positions by maximizing the total surplus (i.e. minimizing total costs). The payment rule is such that each winning bidder pays for the externality exerted on the other bidders by holding a certain position, which excludes her own bid. Under this mechanism it is a weakly dominant strategy to report truthfully (Krishna, 2010), i.e. in this case true outage and backup costs. This renders the VCG mechanism for priority supply a worthwhile approach as it ensures the socially optimal allocation of priority positions and thus improves the resilience of the power system not only in the short term but also in the long term.

The paper is structured as follows: In section 2 the background and problem are described. Section 3 first introduces the set-up and the initial situation with arbitrary assignment of network users to priority positions. Then the optimal assignment is derived. Further, it shows why other auction formats are not suitable for this problem and outlines the Vickrey-Clarke-Groves mechanism. In Section 4 we discuss limitations of the VCG auction and of the particular approach. Section 5 concludes the paper.

2. The problem of inefficient network user ranking

This section introduces the concept of resilience in power network systems in a greater detail and explains why the current mechanisms governing supply disruptions lead to inefficient short-run dispatch on the one hand and to inefficient resilience investment of network users on the other.

By resilience of electricity networks, we mean the ability of the electric power system to deal quickly and efficiently with potential large-scale and long-lasting power interruptions (c.f. acatech, 2020). In this sense, it is up to the grid operator to make the grid flexible and resilient. Resilience comprises two aspects: On the one hand, it means that the damage from supply interruptions is minimised and that a rapid restoration of secure power supply can be achieved through flexible adaptation of the system ("soft resilience"). This is illustrated in Figure 1. On the other hand, it is about increasing the robustness of the system in the sense of resistance to possible threats, so that supply interruptions do not occur in the first place if possible ("hard resilience"). Covering the two, Brunekreeft et al. (2023) propose an approach for resilience incentives in the regulation of electricity network operators.

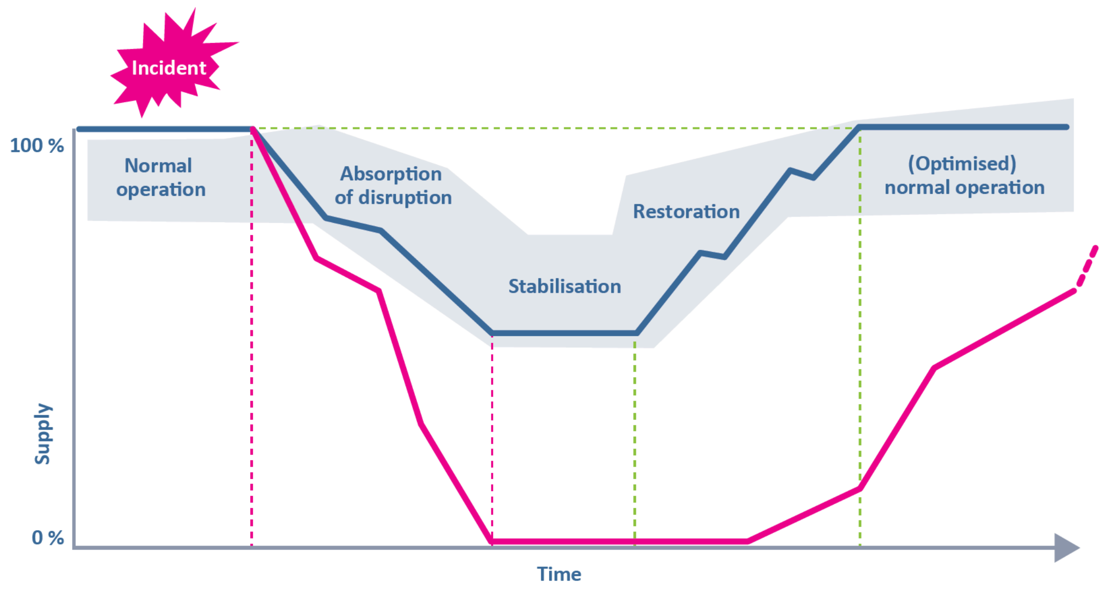

Figure 1: Illustration of resilience (Source: Babazadeh et al., 2018, p. 32.)

In this paper we focus on the first aspect, i.e. soft resilience, in particular on the minimization of outage costs in the case of a disruption. Because network users differ both in the costs that they incur during an outage (c.f. De Nooij et al., 2007) and in the costs incurred for backup measures, the central question addressed here is in which order should network users be disconnected from the grid in case of a disruption? Or similarly, in which order should they be reconnected after a (partial) breakdown of the system?

We propose in the following that this can be achieved by revising the ranking of network users with regard to priority supply according to their outage costs. Because the ranking drives the resilience investment of network users, this ensures also that outage costs decrease in the long-run. For the network operator the main challenge of this task is asymmetric information, because the outage and backup costs of the network users are private information. For the network user the benefits of resilience increasing back-up measures depend on the probability of outages and is therefore rank-dependent. We assume that, initially, network users are arbitrarily assigned to priority positions. In this situation, users who are ranked high in the list and know that they are disconnected last face a low probability of outage and may not invest in backup. Though individually rational, this behaviour might be inefficient from a system perspective, if the backup investment combined with a reassignment to a lower position would reduce total outage costs and, hence, increase resilience of the entire system. A simple numerical example illustrates the problem. Assume two network users A and B that have costs in the event of an outage of 40 and 60 respectively, which could be halved by backup investment. Assume furthermore that investment costs are 5 for user A and 20 for B, and that user A arbitrarily holds a higher position and has an expected outage probability of 0.1, whereas user B has one of 0.5. In this setting neither user would invest in backup and expected total costs would sum up to 34. Yet, if the users were to exchange positions and hence outage probabilities, user A would invest in backup and expected total costs including backup costs would sum up to only 21. Without a reassignment of the users this would lead to a distortion of investment into resilience (of user A in this case) and, hence, to welfare losses.

In practice, in the event of a crisis, the supply of critical energy needs of the population and the fulfilment of public tasks enjoy priority supply. Beyond these areas, the legal requirement is that network users are treated equally (anti-discrimination), e.g. by using random or proportional rationing. Network operators do, nonetheless, have a certain leeway to prioritize consumers. The German Ministry of Economics (BMWK), for instance, states that in the event of a supply crisis "[t]here is no separate legal claim for individual consumers/customer groups to priority supply per se. However, in the event of a crisis - within the framework of technical feasibility - there is the possibility of corresponding prioritisation on the part of the network operators and authorities." (BMWK, 2022, p. 28, own translation). Because no further information is known to us about on which grounds single consumers or consumer groups may be prioritized, a certain inefficiency has to be assumed when it is not based on outage cost information.

The literature on priority service shows, however, that prioritizing electricity customers in the case of service disruptions is more efficient than proportional or random rationing (e.g. Chao & Wilson, 1987; Oren & Doucet, 1990; Noussair & Porter, 1992). The authors propose a limited number of classes for priority or even just two (priority / non-priority), and prices, interruption schemes (c.f. Oren & Doucet, 1990) or auctions (cf. Noussair & Porter, 1992) as a means of implementation. This strand of literature does not consider externalities between the users, as priority positions are not limited. This means that whether one network user is in a certain priority class or not does not affect another user's ability to also be in this class.

In contrast, we assume that priority places are limited. More precisely, we look at the case where each position can only be held by one network user. The consideration behind it is that if load shedding needs to be done, there will be a certain shedding sequence of (large) consumers or distribution nodes that can be aggregated as single consumers. Conversely, when the network is restored after a disruption there is also an order in which single consumers or network nodes are reconnected to the grid. Hence, we need to differentiate several positions that differ in outage probabilities.

The problem of assigning objects to agents maximizing the total value, where one object can only be assigned to one agent and each agent cannot be assigned more than one object, is known as a matching or assignment problem (c.f. Shapley & Shubik, 1971). This problem can be solved by linear programming or other algorithms and results in an optimal ranking when the true values of the agents are known. To reveal these values, position auctions have been developed, which are used among other things for the sale of positions for advertisement on websites of search engines (e.g. Varian, 2007 & 2009; Edelman & Ostrovsky, 2007; Edelman et al., 2007). We build on this strand of literature to develop an auction design for priority positions.

In contrast to the position auction literature that we are aware of we assume that there is an initial, arbitrary assignment of the network users to the priority positions, i.e. network users do not start on equal terms, but may also lose due to a new, lower position. Furthermore, we assume that the agent's values are position dependent due to the possibility of backup investment that affects outage costs. We outline this set-up in more detail in the next section.

3. Auction of priority supply

In the following, we first outline the set-up and the initial assignment of the network users to priority positions. Then we introduce the optimal reassignment of the priority positions by means of a numerical example as a benchmark based on complete information. Subsequently, we outline a Vickrey-Clarke-Groves position auction design for the reassignment of the network users based on this example.

3.1. Set-up and initial situation

We assume N network users $n = 1, 2, ..., N$ that are initially ranked arbitrarily (or by non-economic categories) on a priority list. The ranking contains Y positions $k = 1,2, ..., Y$, with $Y = N$, which include K $(< Y)$ priority positions with ascending outage probabilities p_k $(p_1 < p_2 < ... < p_K)$ and a residual category, *r*, with $(N - K)$ identical positions and the highest outage probability $(p_r >> p_K)$ for the remaining network users. The probabilities are fixed and commonly known. Each priority position can only be assigned to one user, whereas the residual category hosts all remaining users.

Each network user has individual outage costs, OC_n. The network user can decide to invest in backup, which reduces her outage costs to $OC_n^I (< OC_n)$, but induces individual backup costs (C_n). The decision variable of the network user for investing in backup is assumed to be binary $(I_{n,k} \in [0; 1])$.[2] The network user invests in backup capacity in a certain position k, if backup investment in this position is profitable, i.e. when forgone expected outage costs in this position exceed investment costs. As a consequence of the position specific investment decision, the outage costs of a network user are position-dependent, i.e.

[2] Alternatively, investment could be modelled as a continuous variable, i.e. network users vary the size of the storage. We leave this for further research.

$$OC_{n,k} = \begin{cases} OC_n, & \text{if } I_{n,k} = 0 \\ OC_n^I, & \text{if } I_{n,k} = 1 \end{cases} \quad (1)$$

Outage costs with backup in place are of course lower than outage costs without ($OC_n^I < OC_n$). In fact, outage costs are constant across all positions when backup investment is either beneficial in every position or inefficient in any position. In all other cases, it is a step function with one step, i.e. it is constant across a certain number of high positions and then decreases abruptly to the lower value in one position where backup becomes profitable, to remain constant across all other lower positions.

This in turn applies also for the total costs that include the backup investment costs of network user n in position k

$$TC_{n,k} = OC_{n,k} + I_{n,k} \cdot C_n \quad (2)$$

We further assume that the network users are initially randomly assigned to the priority positions, i.e. each network user holds a starting position s. To simplify, we assume that no network user has an incentive to invest in backup in her starting position ($I_{n,s} = 0$ for all n).[3]

3.2. Optimal reassignment of network users with a numerical example

An optimal reallocation of the priority positions to the network users minimizes the sum of expected outage and investment costs of all network users. This problem, where each position can only be assigned to one agent and each agent can only hold one position, is known as an assignment game and can be solved by linear programming or other algorithms (cf. Shapley & Shubik, 1971). A necessary precondition is that the true costs of each user are known. We go into more detail on how to incentivize network users to reveal their true costs and benefits in Section 3.4. Before doing so, let us first illustrate the problem and its optimal solution as a benchmark with a numerical example.

The initial situation is the following: we assume 10 network users ($A, B, \dots, J$) and 5 priority positions ($1, 2, \dots, 5$) with ascending outage probability from 0.1 to 0.5 (increasing in 0.1 steps). The outage probability in the residual category is assumed to be 0.8. The initial ranking, the outage costs and backup costs were randomly chosen and are given in Table 1. As stated above, we assume an initial set-up in which no network user has incentives to invest in backup capacity on her starting position ($I_{n,s} = 0$). Hence, the expected total costs in this situation are only expected outage costs and sum up to 276.6.

[3] This corresponds to the assumption that some network users may have invested in backup measures in the past on the starting position, but would not invest in additional backup given the individual outage and investment costs.

Table 1: Initial ranking and expected costs of bidders

n	$k = s$	p_k	$OC_n = OC_{n,s}$	OC_n^I	C_n	$I_{n,s}$	$E(TC_{n,k}) = p_k \cdot OC_{n,k} + C_n \cdot I_{n,k}$
A	1	0.1	75	37.5	11	0	7.5
B	2	0.2	61	30.5	21	0	12.2
C	3	0.3	53	26.5	15	0	15.9
D	4	0.4	1	0.5	9	0	0.4
E	5	0.5	46	23	23	0	23
F	r	0.8	35	17.5	18	0	28
G			50	25	21	0	40
H			80	40	35	0	64
I			40	20	17	0	32
J			67	33.5	27	0	53.6
							∑=276.6

Before reassigning the positions, the expected total costs need to be calculated for each network user for each position, considering whether or not investment in backup capacity is efficient in the respective position. For network user A, for instance, who would not invest in backup in the starting position 1, investment in backup becomes profitable in position 3 and lower due to the higher expected outage cost savings in these positions. Table 2 shows the expected total costs (sum of expected outage costs and investment costs) of the 10 network users in each position.

Table 2: Expected total costs of the network users in each priority position 1-5 and in the residual category (bold: result of the assignment problem)

	$E(TC_{n,k})$ of									
k	A	B	C	D	E	F	G	H	I	J
1	7.5	6.1	5.3	0.1	4.6	3.5	5	**8**	4	6.7
2	15	12.2	10.6	0.2	9.2	7	10	16	8	**13.4**
3	22.25	**18.3**	15.9	0.3	13.8	10.5	15	24	12	20.1
4	26	24.4	21.2	0.4	18.4	14	**20**	32	16	26.8
5	29.75	30.5	26.5	0.5	**23**	17.5	25	40	20	33.5
r	**41**	45.4	**36.2**	**0.8**	36.8	**28**	40	64	**32**	53.6

The assignment problem is solved by minimizing the expected total costs of all users. Formulated in terms of a linear programming optimization problem, the objective is to minimize

$$min\ T = \sum_k \sum_n E(TC_{n,k}) \cdot X_{n,k} \qquad (3)$$

by choosing the non-negative variable $X_{n,k}$, where $X_{n,k,}$ is the share of position k assigned to network user n. If it is zero the position is not assigned to user n; if it is one the position is fully assigned to user n. The minimum value of T is attained with all $X_{n,k}$ equal to either zero or one.

The problem is subject to the following constraints: each priority position can only be assigned to one network user

$$\sum_{n} X_{n,k} = 1\,,\; for\ \mathrm{k=1,2,...,5} \tag{4}$$

This constraint differs slightly for the residual category r, which is assigned to the remaining five network users that are not assigned to a priority position

$$\sum_{n} X_{n,r} = 5 \tag{5}$$

Furthermore, each network user can only hold one position

$$\sum_{k} X_{n,k} = 1\,,\; \forall n \tag{6}$$

Solving this assignment problem yields the total cost minimizing ranking of network users (c.f. Shapley & Shubik, 1971).

For the given numerical example, the assignment problem was solved in Excel with the Simplex LP engine. The results of the assignment problem are given in Table 3, i.e. the optimal ranking, expected outage and investment costs and resulting expected total costs. Due to the non-linearity of outage and investment costs of a network user across positions (i.e. they vary between positions dependent on the investment decision), this approach does not necessarily assign a position to the user with the highest expected costs in this position (e.g. position 2 assigned to bidder J, not to bidder A, compare Table 2 and Table 3). The sum of expected total costs of the optimal assignment are 220.7, which is 55.9 lower than the initial allocation.

Table 3: Optimal assignment based on true costs (solve by Simplex LP minimizing expected total costs),

k	n	$E(OC_{n,k})$	$I_{n,k} \cdot C_n$	$E(TC_{n,k})$
1	H	8	0	8
2	J	18.3	0	18.3
3	B	13.4	0	13.4
4	G	20	0	20
5	E	23	0	23
r	A	30	11	41
	C	21.2	15	36.2
	D	0.8	0	0.8
	F	28	0	28
	I	32	0	32
				∑=220.7

The reassignment of priority positions has the effect that two network users (A & C) would invest in backup, which they would not have done in their starting position. Although the overall effect would be a reduction in total costs, some network users would benefit from the optimal reassignment whereas others would lose or not experience any effect. Table 4 shows the effect the reassignment would have on each network user, i.e. the difference in expected total costs of the initial and the new position.

Table 4: Expected total costs of optimal assignment for each network user compared to initial situation

k	n	s	$E(TC_{n,s}) - E(TC_{n,k})$
1	H	r	56
2	J	r	40.2
3	B	2	-6.1
4	G	r	20
5	E	5	0
r	A	1	-33.5
	C	3	-20.3
	D	4	-0.4
	F	r	0
	I	r	0
			∑=55.9

Note that these effects result, if the reassignment was solved based on complete information and without any payments by the network users.

3.3. Suitability of commonly used position auction mechanisms

The information about the total costs of each position is private and non-observable by the network operator. In order to reduce expected total outage costs the network operator needs to find a way to reveal this information. An established way to do so is by setting up an auction, in this case a position auction.

In position auctions, e.g. for positions with descending attractiveness in online advertisement, each bidder typically enters one single-dimensional bid, which states her position independent valuation (e.g. value per click), without specifying which position they are bidding for. The assumption is that the value of each position (e.g. click rate) is shared public information. The allocation rule is to assign the highest position to the bidder with the highest bid, the second highest position to the bidder with the second highest bid, and so on. The allocation is efficient in this setting, if the highest position is assigned to the bidder not only with the highest reported bid, but who values it the most, and respectively for the other positions. The crucial question is therefore whether bidders have incentives to provide truthful information, or at least whether they all deviate from their true value to the same extent and in the same direction, so that an efficient ranking nevertheless results when this allocation rule is applied. The bidding incentives are given by the payment rule applied. Among the most commonly used payment rules in position auctions are the generalized first-price (GFP) and the generalized second-prize (GSP) auction (c.f. Edelman et al., 2007). In GFP auctions, the successful bidder pays the amount of her own bid, which gives the incentives to bid below the true valuation (c.f. Krishna, 2010; Edelman et al., 2007). Each bidder will try to guess the next highest bidder's bid and will bid some increment higher. Inefficiencies result under this rule when the competitors' bids are misjudged.

In GSP auctions, adapted from the second price rule for single unit auctions developed by Vickrey (1961), each bidder pays the next highest bid to her own bid for the new position. In single-unit second-price auctions, this induces truthful bidding (cf. Vickrey, 1961). For GSP it has been shown, however, that this is not necessarily the case and that this design may also result in inefficiencies (c.f. Edelman & Ostrovsky, 2007; Varian & Harris, 2014). We will go into more detail on this aspect below.

In short, even in standard position auctions the two designs do not reliably lead to efficient allocations. In the following, we outline why these two auction designs are not suitable for the given particular context and why the Vickrey-Clarke-Groves mechanism is.

First of all, the allocation rule in the given context differs from the standard auction format, because a priority position does not always go to the bidder with the highest reported valuation, as outlined in Section 3.2. The reason for this is the nonlinearity of the investment and outage costs of the network users and thus of the private valuation of the priority positions. A standard auction would not lead to an efficient outcome, even if it was based on true values, unless resale was possible. The more complex allocation rule makes it less obvious for the bidders to estimate who is the direct competitor for a certain position, which requires not only to estimate the outage costs of the competitors, but also who will invest on which position. In GFP auctions, where misjudgements of the competitors' bids may already occur in standard auction formats, they are even more likely in settings where it is not clear who the next highest bidder is.

In GSP auctions, as indicated above, bidders may have an incentive to deviate from truthfully bidding and to strategically underbid bidders with lower values, resulting in inefficiencies (c.f. Edelman & Ostrovsky, 2007). This can be the case when the decrease in value (here increase in the expected total costs) of being in a lower position is outweighed by an even greater decrease in payment such that the net benefit of being in the lower position is higher. For the numerical example it can be shown that this is the case (see Section 3.4, footnote 6). When, additionally, the bidders misjudge their competitors' bids, even greater inefficiencies may result due to this bidding behaviour.

Overall, the required more complex allocation mechanism needed in this context due to the nonlinearities of total costs challenges auction designs in which the bidders' bids depend on the (estimated) bids of the competitors. The probability of under- or overestimating the next highest bid is likely to increase when it is not obvious who the next highest bidder is, resulting in inefficient allocations.

In contrast, the Vickrey-Clarke-Groves mechanism, with its comparatively complex payment rule that charges each bidder for the externalities it exerts on all other bidders by holding a particular position, can adequately address this challenge. The VCG payment rule not only decouples payment from one's own bid (as in the GSP auction), but also aligns the incentives of the single bidder with the total welfare of all bidders by internalizing the external effects on others. It makes each bidder's strategy independent of the likely actions of the competitors (dominant strategy property) (Ausubel & Milgrom, 2006). As a consequence, the bidders do not need to learn about their competitors' values and strategies and therefore cannot misjudge them. Therefore, the mechanism reliably leads to efficient allocations. For these reasons we propose to address this problem by means of the Vickrey-Clarke-Groves (VCG) mechanism, which we outline in the following.

3.4. Vickrey-Clarke-Groves position auction for reassignment of priority positions

We suggest an auction design that applies the Vickrey-Clarke-Groves (VCG) mechanism[4] as it induces truthful bidding and can thus effectively solve the information asymmetries between network operator and network users. In the following we outline the mechanism in general and by applying it to the numerical example.

The network users enter a sealed bid for every position ($b_{n,k}$), stating their willingness to pay for each position.[5] Given that the network users enter bids for every position, even the residual category, this willingness to pay can also be negative. Based on these bids, the network operator re-assigns the priority positions and the positions in the residual category to the users (i.e. the bidders in the auction). When assigned to a certain position, the bidder has to make a payment (M_n) according to the specified payment rule. In our case the payment results from the VCG mechanism, which we outline below.

[4] The mechanism is named after William Vickrey, Edward Clarke and Theodore Groves. The central idea that the highest bidder wins but only pays the best losing bid, i.e. the payment is independent of the bidder's own bid, was developed by Vickrey (1961). Clarke (1971) and Groves (1973) generalized Vickrey's results.

[5] The sealed bid format not only has practical advantages, i.e. network users do not have to come together in one location at the same time (c.f. Krishna, 2010), but it can also prevent collusion of the losing bidders (c.f. Klemperer 2002a, 2002b).

The benefit of the auction to network user n, i.e. the benefit of the new position is the difference in expected total costs in the new position k compared to the initial position s net of the payment,

$$B_{n,k} = E(TC_{n,s}) - E(TC_{n,k}) - M_n \tag{7}$$

The first two arguments on the right-hand-side of this equation is bidder n's private value of position k and private information.

$$v_{n,k} = E(TC_{n,s}) - E(TC_{n,k}) \tag{8}$$

Recall that the expected total costs vary between the positions not only due to the difference in outage probabilities, but also, potentially, due to different backup investment decisions (see eq. (1) & (2)). The value is positive for a higher position than the starting one, zero for the starting position and negative for a lower position. The payment in the VCG mechanism, M_n, is independent from the bidder's own bid. The principle behind the mechanism is to charge each bidder the negative externality that she imposes on other bidders by holding a certain position. In other words, it compares the auction result for all bidders ranked below the respective bidder with the outcome that would result, if the bidder did not participate at all and the lower positioned bidders would have been assigned to one position higher each. In the given example the externality that bidder n assigned to position k imposes on the other bidders is the difference between the total revealed benefits of all bidders except the respective bidder of the assigned position and the total revealed benefits of all these bidders of the assigned position that would result, if the bidder was absent.

Based on the bids of the bidders for each position the network operator computes a value-maximizing allocation given the allocation rule X of the bids (and thus implicitly the bidders) to the positions (c.f. Krishna & Perry, 1998; Krishna, 2010)

$$X^*(b) \in arg\ max_{X_{1,1},\dots,X_{N,Y}} \sum_{m} \sum_{l} b_{m,l} X_{m,l} \tag{9}$$

Subject to the restrictions of eq. (4)-(6). As a result, the variable $X_{m,l}$ either takes a value of 1 (if position l is assigned to m) or of 0 (if not). In contrast, $X^*(b_{-n})$ is an efficient alternative that would result if bidder n was not present, i.e.

$$X^*(b_{-n}) \in arg\ max_{X_{1,1},\dots,X_{N,Y}} \sum_{m \neq n} \sum_{l} b_{m,l} X_{m,l} \tag{10}$$

Subject to the same restrictions of eq. (4)-(6). The payment in the VCG mechanism is determined by the difference between the values resulting from two allocations, with the crucial exclusion of the bids of bidder n (c.f. Ausubel & Milgrom, 2006; Krishna & Perry, 1997)

$$M_n(b) = \sum_{m \neq n} \sum_{l} b_{m,l} \left(X^*_{m,l}(b_{-n}) - X^*_{m,l}(b) \right) \quad (11)$$

Hence, M_n is the externality that bidder n exerts on the other bidders by holding position k: it sums for each of the other bidders the difference in reported values of the alternative position the bidder would be assigned to, if n had not been not present, and of the position the bidder is actually assigned to. Put differently, the bidder pays the sum of values of all others in the event that she had been absent, but receives the sum of values of all others for the final allocation. A simple numerical example illustrates. Let us assume 3 bidders (A, B and C) and two priority positions (1 and 2). The bidders report the following values for position 1 (from A to C): 10, 9, 8 and for position 2: 8, 7, 6. The value maximizing allocation based on these bids is to assign position 1 to bidder A and position 2 to bidder B. If bidder A was not present, the other bidders would all move up one position. Hence, the VCG payment for bidder A is the sum of the externalities exerted on B and C: $M_A = b_{B,1} - b_{B,2} + b_{C,2} = 9 - 7 + 6 = 8$. The payment is independent from the bids that bidder A reported.

Because the bidders ranked higher than k are not affected by the presence of bidder n on this position, eq. (11) can also be reduced to

$$M_n(b) = \sum_{m \neq n} \sum_{l \geq k} b_{m,l} \left(X^*_{m,l}(b_{-n}) - X^*_{m,l}(b) \right) \quad (12))$$

As a consequence of this rule, bidders ending up in the residual category pay nothing. For auctions using this payment rule, it has been shown that it is a weakly dominant strategy to bid truthfully (e.g. Krishna, 2010, Ausubel & Milgrom, 2006; Edelman & Ostrovsky, 2007), i.e. in our case the bidders bid the true value for each position, $b_{n,k} = v_{n,k}$.This is because the bidder's own bid does not affect the price she pays, but only determines whether she wins a certain position or not. The price is determined exclusively by the competing bids. Only when the bidder bids truthfully, she can be sure to win when she is willing to pay the price.

In the numerical example, the values of the network users of each position, i.e. the difference in expected total costs compared to the starting position, are given in Table 5**Error! Reference source not found.**.

Table 5: Private value of each position of each network user

k	$v_{n,k}$ of									
	A	B	C	D	E	F	G	H	I	J
1	0	6.1	10.6	0.3	18.4	24.5	35	56	28	46.9
2	-7.5	0	5.3	0.2	13.8	21	30	48	24	40.2
3	-14.75	-6.1	0	0.1	9.2	17.5	25	40	20	33.5
4	-18.5	-12.2	-5.3	0	4.6	14	20	32	16	26.8
5	-22.25	-18.3	-10.6	-0.1	0	10.5	15	24	12	20.1

r	-33.5	-33.2	-20.3	-0.4	-13.8	0	0	0	0	0

Bidding truthfully implies that the network users reveal these values in their bids. The positions are then allocated maximizing the sum of revealed position values, which minimizes total costs. The resulting ranking, payments and benefits are given in Table 6. The ranking is the same as in the optimal assignment, whereas the effect (benefit) for each network user differs due to the payment (compare to Table 4).

Table 6: Payments and benefits under VCG mechanism

n	k	$b_{n,k} = v_{n,k}$	M_n	$B_{n,k} = v_{n,k} - M_n$
H	1	56	34.4	21.6
J	2	40.2	27.7	12.5
B	3	-6.1	21.6	-27.7
G	4	20	16.6	3.4
E	5	0	12	-12
A	r	-33.5	0	-33.5
C		-20.3	0	-35.3
D		-0.4	0	-0.4
F		0	0	0
I		0	0	0
		∑=55.9	∑=112.3	∑=-56.4

Due to the position-dependence of the total costs, the total surplus maximizing allocation again does not necessarily allocate the bidder with the highest respective bid to a certain priority position. In the numerical example this can be seen in the assignment of priority position 3: as shown in Table 5**Error! Reference source not found.**, it is bidder G who enters the highest bid ($b_{G,3} = v_{G,3} = 25$) for this position (except for the bids of the already assigned bidders H & J). The position is, however, assigned to bidder B instead (see Table 6). The reason for this is that the increase in expected total costs of bidder B moving down to position 4 outweighs the reduction in expected total costs of bidder G from moving up to position 3 (which would be $-6.1 + 5 = -1.1$ in this case, see Table 5**Error! Reference source not found.**).

The result is a Nash equilibrium, if the bidders do not have an incentive to win either a higher position (by overstating their true value) or a lower position (by understating their true value). For this it must hold that $B_{n,k} > B_{n,k'}$ for all $k \gtrless k'$ and for all n.

For the numerical example it can be shown that this holds, at least for $k' = k + 1$ and $k' = k - 1$. Table 7 gives the resulting payments and benefits if the bidders underbid (moving one position down) or overbid (moving one position up), compared to the truthful bid. If we consider, for example, that bidder H underbid bidder J and would be assigned to position 2 instead of position 1, the benefit for H would decrease by 1.3 due to a decrease in value by 8 and a (respectively lower) decrease in payment by 6.7 (see third row of Table 7). Hence, bidder H does not have an incentive to understate her value.[6] The same is true for all bidders whether they over- or underbid their true values for a position. Edelman et al. (2007) call such an equilibrium "locally envy-free".[7] That truthful bidding is a weakly dominant strategy in general under the VCG mechanism was shown, e.g. by Ausubel & Milgrom (2006) and Edelman & Ostrovsky (2007). Ausubel & Milgrom (2006) further showed that when each bidder bids truthfully, the outcome maximizes total value.

Table 7: Payments and benefits under VCG mechanism (if bidders bid truthfully, underbid or overbid their true WTP), red: lower benefit than under truthful bidding)

n	Truthful bidding ($b_{n,k} = v_{n,k}$)				Underbidding ($b_{n,k} < v_{n,k}$)				Overbidding ($b_{n,k} > v_{n,k}$)			
	k	$v_{n,k}$	M_n	$B_{n,k}$	$k+1$	$v_{n,k+1}$	M_n	$B_{n,k+1}$	$k-1$	$v_{n,k-1}$	M_n	$B_{n,k-1}$
H	1	56	34.4	21.6	2	48	27.7	20.3	n/a			
J	2	40.2	27.7	12.5	3	33.5	21.6	11.9	1	46.9	35.7	11.2
B	3	-6.1	21.6	-27.7	4	-12.2	16.6	-28.8	2	0	28.3	-28.3
G	4	20	16.6	3.4	5	15	12	3	3	25	22.7	2.3
E	5	0	12	-12	r	-13.8	0	-13.8	4	4.6	17	-12.4
A	r	-33.5	0	-33.5	n/a				5	-22.25	13.8	-36.05
C	r	-20.3	0	-20.3					5	-10.6	13.8	-24.4
D	r	-0.4	0	-0.4					5	-0.1	13.8	-13.9
F	r	0	0	0					5	10.5	13.8	-3.3
I	r	0	0	0					5	12	13.8	-1.8

Summing up, a position auction based on the VCG mechanism replicates the ranking of the optimal reassignment of the network users to the priority positions in the numerical example, as it induces truthful bidding. This design, hence, minimizes total expected outage and investment costs and incentivises optimal backup investment by the network users.

4. Discussion

We have shown in section 3 that the VCG mechanism is suitable for the position auction of priority supply. At the same time, it has some limitations, which we discuss in this section for

[6] Note that this would not be the case in a GSP auction design. Under the GSP payment rule bidder H, if she bid truthfully, would have to pay the second highest bid for position 1, i.e. 46.9 (bidder J's bid). Her benefit would be 56-46.9=9.1. Now assume that bidder H underbids J instead and is assigned to position 2. Her valuation of position 2 is lower of course (48 instead of 56), yet the difference in payment is even greater (30 instead of 46.9). The benefit of being in position 2 under GSP for bidder H is therefore greater than of being in position 1 (18 instead of 9.1).

[7] It should be noted that for this definition Edelman et al. (2007) focus on whether a bidder could increase her payoff, if she would change with the bidder ranked above her (by overbidding the true valuation) and do not consider the incentives for underbidding the true valuation.

the given context. Furthermore, we address the issues of the loss of losing bidders and the estimation of outage probabilities, which are specific for this context.

4.1. Limitations of the VCG mechanism

One drawback commonly connected with the VCG auction is the revenue deficiency, i.e. it leads to lower revenues than other auction designs (c.f. Green & Laffont, 1979), and the non-monotonicity of seller revenues (c.f. Ausubel & Milgrom, 2006). In the given context this can be seen as a less crucial problem, as the main aim of the seller in this case (i.e. the network operator) is to reduce the expected outage costs of the network users and to incentivize efficient backup investment, which is achieved. On this backdrop it needs to be assured that the regulatory incentives of the network operator align with this aim. From the network users' perspective, the lower payments in contrast to other auction designs can be seen as an advantage.

Another critical aspect of a VCG auction mentioned in the literature is the vulnerability to collusion of losing bidders (c.f. Ausubel & Milgrom, 2006). Whether this is a risk in this context as well, depends on further aspects such as the number of (losing) bidders, on the frequency of repetition and the details of the auction design (e.g. whether the bids are sealed) (c.f. Klemperer 2002a, 2002b). The risk of collusion can be assumed to be low, if the number of losing bidders is high, the frequency of repetition low and when the bids are sealed.

A third, more crucial argument in this context might be the one raised by Rothkopf et al. (1990) against the VCG mechanism, namely that bidders may be reluctant to report their true valuations for fear that this revelation could be used against them at a later point in time. In this particular context the information revealed can furthermore be defined as critical, which needs to be protected against unlawful use (e.g. in cyber-attacks). The auctioneer, i.e. the network operator, and the network regulator have to find a balance between transparency and data protection in this case. Because both are used to handle critical information in the regulatory practice, however, this aspect can be considered less critical.

Another stated weakness of the VCG mechanism is indeed a strength in this context, as outlined in Section 3.3: the complexity of the mechanism. On the one hand, it raises the costs of implementation for the auctioneer and for the explanation to the participants (Ausubel & Milgrom, 2006; Varian & Harris, 2014). On the other hand, it reduces the costs of participation for the bidders because they save expenses on learning about their competitors' values and strategies (Ausubel & Milgrom, 2006). This last aspect renders the mechanism particularly useful in settings with complex allocations rules such as the one presented in this article.

4.2. Loss of losing bidders and outage probabilities

In contrast to the standard auction, the bidders with the lowest feasible valuation do not necessarily expect zero surplus, but may also expect a loss, as they may be downgraded and experience an increase in total costs. This results not from the payment scheme but from the assumption of an initial arbitrary assignment of the network users to priority positions. One can argue that because the first assignment was arbitrary, the loss experienced by the reassignment is acceptable, as the network users enjoyed unjustified privileges on the previous position. Yet, to increase the acceptability of the position auction by the network users the distributional effects could be addressed by side payments. Such payments have to be carefully designed in order to not distort the incentives for truthful bidding.

In our approach we assumed that the outage probabilities are known and constant. A practical test is needed to determine whether the outage probabilities can be defined precisely enough to set-up a position auction. Furthermore, various developments may have an effect on these probabilities over time, such as resilience investments by the network operator. Major changes may be a reason to repeat the auction.

5. Conclusion

This paper outlines an auction format for priority supply during supply disruptions to increase the resilience of the electricity system. We understand resilience as the ability of the electric power system to deal quickly and efficiently with potential large-scale and long-lasting power interruptions and focus here on the “soft” facet of resilience, i.e. the reduction of damage caused by supply disruptions.

One way to reduce damage costs, when curtailment is inevitable, is to cut off first the network users that have the lowest damage and/or back-up costs. The central problem to realize such a ranking of network users is asymmetric information: the network operators do not have the information needed to prioritize network users efficiently in the short term. The network users, on the other hand, can increase the system’s resilience in the long-term by means of back-up measures. Their decision on back-up measures will be based on expected outage probabilities and thus on expected outage costs. If the probabilities result from an inefficient prioritization method, also the backup will be inefficient.

The commonly used designs in position auctions are not well suited to address this problem as they do not guarantee efficient results in standard auction formats and even less likely in the case when there are non-linear valuations of priority positions. Therefore, we propose to apply the Vickrey-Clarke-Groves mechanism to solve the problem. The network operator publishes the number of priority positions and the associated outage probabilities. The network users have to enter bids for each position, otherwise they remain in or are assigned to the residual category with the highest outage probability.

The payment principle of the VCG mechanism is that winning bidders pay the externality that they impose on other bidders by holding a certain position, excluding their own bid, which ensures truthful bidding and at the same time relieves the bidders from estimating and learning about their competitors' values and strategies. Using numerical examples, we argue that the VCG mechanism replicates the optimal assignment of network users to priority positions and leads to efficient back-up investments, thus minimizing total outage costs. Although ensuring the socially optimal allocation of priority positions, the mechanism has some practical limitations. Central in this particular context may be the reluctance of the network users to reveal their true costs for fear of subsequent repercussions.

Aware of these limitations, we discuss practices from the online advertisement auctions that address these problems. The implementation of the VCG mechanism in the online advertisement context promises that the practical limitations can be addressed sufficiently and suggests that the mechanism would also work in practice for position auctions of priority supply. The details of implementation, such as the determination of outage probabilities for each position is an issue for further research. In sum, implementing VCG for priority supply during disruptions is worthwhile as it improves the resilience of the power system not only in the short-run but also in the long-run.

6. References

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